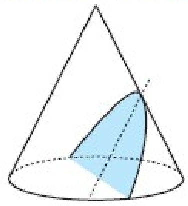
Focus on Quadratic functions and parabolas :

1/ Introduction :

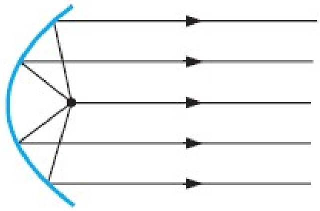
The graphs of all quadratic functions are **parabolas**. The parabola is one of the conic sections.

Conic sections are curves which can be obtained by cutting a cone with a plane. The Ancient Greek mathematicians were fascinated by conic sections.



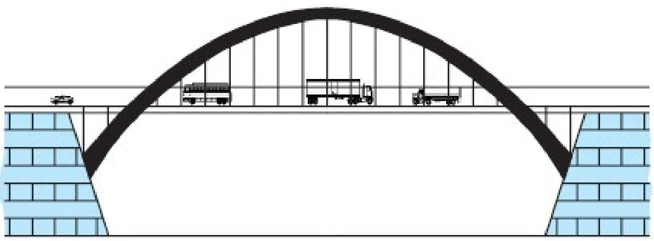
You may like to find the conic sections for yourself by cutting an icecream cone. Cutting parallel to the side produces a parabola, i.e.,

There are many examples of parabolas in every day life. The name parabola comes from the Greek word for **thrown** because when an object is thrown its path makes a parabolic shape.



Parabolic mirrors are used in car headlights, heaters, radar discs and radio telescopes because of their special geometric properties.

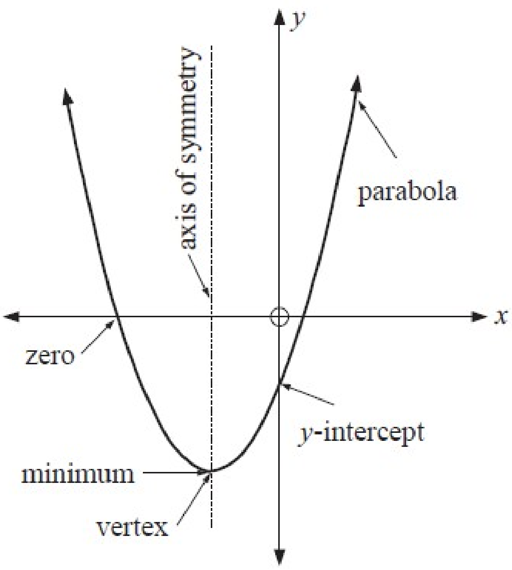
Alongside is a single span parabolic bridge.



Some archways also have parabolic shape.

2/ Review of Terminology

The equation of a **quadratic function** is given by $y = ax^2 + bx + c$, where $a \neq 0$.



The graph of a quadratic function is called a **parabola**. The point where the graph 'turns' is called the **vertex**.

If the graph opens upward, the y coordinate of the vertex is the **minimum** (concave up), while if the graph opens downward, the y-coordinate of the vertex is the **maximum** (concave down).

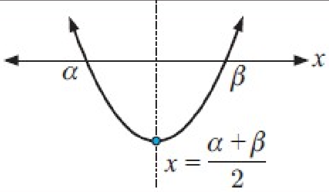
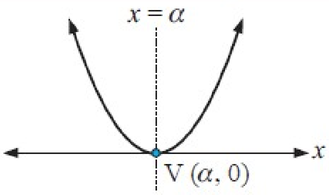
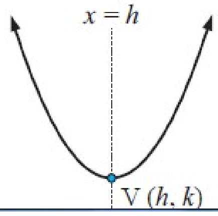
The vertical line that passes through the vertex is called the **axis of symmetry**. All parabolas are symmetrical about the axis of symmetry.

The point where the graph crosses the y-axis is the **y-intercept**.

The points (if they exist) where the graph crosses the x-axis should be called the **x-intercepts**, but more commonly are called the **zeros** of the function.

3/ Sketching a parabola

If the coefficient of x^2:
- is positive, the graph is concave up  $a > 0$
- is negative, the graph is concave down  $a < 0.$

Quadratic form, $a \neq 0$	*Graph*	*Facts*
• $y = a(x-\alpha)(x-\beta)$ α, β are real		x-intercepts are α and β axis of symmetry is $x = \frac{\alpha+\beta}{2}$
• $y = a(x-\alpha)^2$ α is real		touches x-axis at α vertex is $(\alpha, 0)$ axis of symmetry is $x = \alpha$
• $y = a(x-h)^2 + k$		vertex is (h, k) axis of symmetry is $x = h$

4/ Algebra and Calculus

- To solve the equation $f(x) = 0$, find the discriminant : $\Delta = b^2 - 4ac$:

If <u>$b^2 - 4ac > 0$</u>

then the equation $f(x) = 0$ has two distinct **roots** (or **zeroes** or ***x*–intercepts)** :

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{\Delta}}{2a}$$

and $f(x) = a(x-x_1)(x-x_2)$ **(factorized form)**

If <u>$b^2 - 4ac = 0$</u>

then the equation $f(x) = 0$ has only one root $x_0 = \frac{-b}{2a}$

and $f(x) = a(x-x_0)^2$

If <u>$b^2 - 4ac < 0$</u> then the equation $f(x) = 0$ has no (real) roots and can't be factorized.

- <u>To find Maximum or minimum values :</u>

derivative of a quadratic function $f'(x) = 2ax + b$